Close Wednesday: HW_3A,3B,3C
(complete sooner!)
Office hours 1:30-3:00pm in Smith 309
Exam 1 is Thursday in normal quiz section. Covers 4.9, 5.1-5.5, 6.1-6.3.

Entry Task:
Consider the region, *R*, bounded by
$y = \sin(x^2)$, $y = 0$, $x = 0$, and $x = \sqrt{\pi}$.
Find the volume of the solid obtained by rotating *R* about the *y*-axis.

Example: Consider the region, *R*, bounded by

$$y = x^2 - x^3, x = 1, \text{and } y = 0.$$

Set up the integral for the volume of the solid obtained by rotating R about…

(a) …the *x*-axis
(b) …the *y*-axis.
(c) …the vertical line $x = -3$.
(d) …the vertical line $x = 4$.
(e) …the horizontal line $y = 5$.

Flow chart of all Volume of *Revolution* Problems

Step 0: Draw an accurate picture!!! (Always draw a picture)

Step 1: Choose and label the variable (based on the region and given equations)

If *x*, draw a typical **vertical** thin approximating rectangle at *x*.

If *y*, draw a typical **horizontal** thin approximating rectangle at *y*.

Step 2: Is the approximating rectangle perpendicular or parallel to the rotation axis?

$$\text{Perpendicular} \rightarrow \text{Volume} = \int_a^b (\pi(\text{outer})^2 - \pi(\text{inner})^2)(\text{dx or dy})$$

$$\text{Parallel} \rightarrow \text{Volume} = \int_a^b 2\pi(\text{radius})(\text{height})(\text{dx or dy})$$

Step 3: Write everything in terms of the desired variable, fill in patterns, integrate.

The above method is how you should approach the problems, but if you are still having trouble seeing which variable goes with which method here is a summary:

Axis of rotation	**Disc/Washer**	**Shells**
x-axis (or any horizontal axis)	*dx*	*dy*
y-axis (or any vertical axis)	*dy*	*dx*

EXAM 1 IS THURSDAY IN QUIZ SECTION

Allowed:

1. A **Ti-30x IIS Calculator**
2. An 8.5 by 11 inch sheet of handwritten notes (front/back)
3. A pencil or black/blue pen

Details and rules:

1. 5 pages of questions, 80 minutes, use your time effectively.
2. **Show your work using methods from class.** The correct answer with no supporting work is worth zero points. If you guess or use some formula from physics, you don't get credit.
3. Clearly indicate work you want graded. Put a box around your final answers.
4. Leave your answer in exact form, BUT simplify standard trig, inverse trig, natural logarithm, and root values. Here are some examples of un-simplified answers I have seen on tests in the past (I took off one point):

$\sqrt{4} =$, $8^{2/3} =$, $\frac{3}{2} - \frac{2}{5} =$

$\cos(0) =$, $\cos(\pi) =$, $\cos\left(\frac{\pi}{6}\right) =$

$\sin\left(\frac{3\pi}{4}\right) =$, $\tan\left(\frac{\pi}{4}\right) =$, $\tan^{-1}(1) =$

$\ln(1) =$, $\ln(e) =$, $e^0 =$

Quick Review

1. Riemann Sum Approximation
   (Left/Right/Midpoints)
   Riemann Sum Notation

2. Definition of definite integral.
   Definition of indefinite integral

3. Antiderivatives, solving for constants.

4. Fund. Thm. of Calculus, part 1.

5. Fund. Thm. of Calculus, part 2.

6. Net Change and Total Change,
   distance/velocity/acceleration

7. Substitution.

8. Areas between Curves.

9. Volumes of solids:
   - Cross-sectional area method,
   - Cylindrical shells.

<u>6.4 Work</u> **(**Work = "total effort"**)**

The concept "work" measures energy expended in completing a task. When a **constant** force is applied through a fixed distance, we define:

Work = Force · Distance (W = F · D)

If force or distance change in some way during the task (i.e. NOT constant), then we can break up the problem into subtasks, approximate with F · D on each subtask, and add up the approximations.

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} (\text{Force} \cdot \text{Distance})$$

But, to use calculus, we must *find the pattern* for the force and the distance for each subdivision.

But first, some units:

Recall Newton's second law

Force = Mass· Acceleration (F = m·a)

	Metric	**Standard**
Mass		
Acceleration on Earth		
Force		
Distance		
Work		

PROBLEM TYPE 1: *Force changing*.
(Leaky buckets and springs)
If we are moving an object along a number line from $x = a$ to $x = b$ and $f(x)$ = "force at x", then

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

Examples (of changing force):

1. *Leaky bucket*: A leaking bucket is lifted 10 feet. At the bottom the bucket weighs 120 pounds and at the top the bucket weighs 100 pounds. Assume the water leaked out a constant rate as it was lifted (*i.e.* the force function is *linear*). How much work was done to life the bucket?

2. *Springs*:

A weight is attached to the end of a spring and the other end is attached to the wall. Let *L* be the distance the weight is from the wall when it is at rest. We call this *natural length*.

Hooke's law: Force is proportional to the distance from natural length. That is, for each spring, there is a constant *k* such that

f(x) = kx = force to hold the spring x units from natural length.

(x = 0 corresponds to natural length)

Example: Assume natural length for a given spring is 5 cm from the wall. And you know 5 Joules of work are done to stretch from 5cm from wall to 9cm from wall. How much work is done to stretch from 7cm to 10cm from wall?